

# Unit 1: Trigonometric Functions

Content Area:	Math
Course(s):	MATH ANALYSIS
Time Period:	Marking Period 1
Length:	5/6 weeks
Status:	Published

## Standards

---

### Math Standards

---

MA.F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$ , $\pi/4$ and $\pi/6$ , and use the unit circle to express the values of sine, cosines, and tangent for $\pi - \theta$ , $\pi + \theta$ , and $2\pi - \theta$ in terms of their values for $\theta$ , where $\theta$ is any real number.
MA.F-TF.A.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
MA.F-TF.B.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
MA.G-SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

### Life Literacies and Key Skills

---

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

### Mathematical Practices

---

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.

**Learning Plan / Pacing Guide**

---

Suggested timeline: Honors will complete at a faster pace. Unit test will be after week 4.

**CP**

- Week 1
  - Special right triangles (review)
  - DMS -- Radians
  - Unit circle - Activity - Students will derive the values that exist on the Unit Circle.
- Week 2
  - Sine, cosine, tangent
    - Pythagorean Theorem
    - Reference angles
  - Quiz
- Week 3
  - Trigonometric identities
- Week 4
  - Simplifying trig expressions
  - Quiz
  - Verifying trig identities
- Week 5
  - Review
  - Test
  - Performance Assessment

**HONORS**

- Week 1
  1. 4.1 - Radian and Degree Measure
  2. 4.2 - The Unit Circle
- Week 2
  1. Quiz 4.1-4.2
  2. 4.3 - Right Triangle Trigonometry
- Week 3
  1. 4.4 - Trigonometric Functions of Any Angle
  2. Test 4.1-4.1
  3. 4.5 - Graphs of Sine and Cosine
- Week 4
  1. Continue 4.5
  2. Quiz 4.5
  3. 4.6 - Graphs of Other Trigonometric Functions

#### 4. Review 4.5 and 4.6

- Week 5

1. Graphing Trigonometric Functions Test
2. Trigonometric Art Project
3. 4.7 - Inverse Trigonometric Functions

- Week 6

1. Sinusoidal Regression Project

### **Transfer Goals**

---

### **Transfer Goals**

---

Students will be able to independently use their learning to understand and apply the relationship between the trigonometric functions, angles, triangles, and circles.

### **Concepts**

---

### **Essential Questions**

---

- Describe how do you convert between radians and degrees?
- How is right triangle trigonometry used to solve right triangles?
- How is the unit circle used to describe trigonometric functions?
- What is the purpose of inverse trigonometric functions?
- Why should we study trigonometry?

### **Understandings**

---

- Inverse trigonometric functions can be useful in exploring how two aspects of a real-life problem relate to each other.
- Sine and Cosine functions are often used in scientific calculations.

- You can use trigonometry to analyze all aspects of a geometric figure.

## **Critical Knowledge and Skills**

---

### **Knowledge**

---

Students will know:

- Unit Circle
- Trigonometric Functions: Sine, Cosine, Tangent, Cosecant, Secant and Cotangent
- Inverse Trigonometric Functions: Arcsine, Arccosine and Arctangent

### **Skills**

---

Students will be able to:

- Identify a unit circle and describe its relationship to real numbers.
- Evaluate trigonometric functions of any angle.
- Evaluate inverse trigonometric functions.
- Use trigonometric functions to model and solve real-life problems.

## **Assessment and Resources**

---

### **School Formative Assessment Plan (Other Evidence)**

---

- Homework
- Quizzes
- Exit Tickets
- Reflections
- Performance Tasks

## **School Summative Assessment Plan**

---

- Unit Assessment

## **Primary Resources**

---

- Larsen, Ron, Hostetler, Robert P. and Edwards. Bruce H. *Pre-Calculus with Limits: A Graphing Approach* Houghton Mifflin Company, 2005

## **Supplementary Resources**

---

- IXL
- Kutasoftware
- Desmos
- Khan Academy
- PatrickJMT

## **Technology Integration and Differentiated Instruction**

---

### **Technology Integration**

---

- **Google Products**
  - Google Classroom - Used for daily interactions with the students covering a vast majority of different educational resources (Daily Notes, Exit Tickets, Classroom Polls, Quick Checks, Additional Resources/ Support, Homework, etc.)
  - GAFE (Google Apps For Education) - Using various programs connected with Google to collaborate within the district, co-teachers, grade level partner teacher, and with students to stay connected with the content that is covered within the topic. Used to collect data in real time and see results upon completion of the assignments to allow for 21st century learning.
- **One to One Student's laptop**
  - All students within the West Deptford School District are given a computer, allowing for 21st century learning to occur within every lesson/topic.
- **Additional Support Videos**

- Videos can be assigned from PatickJMT, Khan Academy and Youtube, etc... to support each of the lessons within this topic.

## **Differentiated Instruction**

---

### **Gifted Students (N.J.A.C.6A:8-3.1)**

- ☐ Within each lesson, the Gifted Students are given choice on topic and subject matter allowing them to explore interests appropriate to their abilities, areas of interest and other courses.

### **English Language Learners (N.J.A.C.6A:15)**

- ☐ Within each lesson, the English Language Learners are given choice of topic and resources so that their materials are within their ability to grasp the language.
- ☐ Work with ELL Teacher to allow for all assignments to be completed with extra time.

### **At-Risk Students (N.J.A.C.6A:8-4.3c)**

- ☐ Within each lesson, the at-risk students are given choice of topic and resources so that their materials are within their ability level and high-interest.

### **Special Education Students (N.J.A.C.6A:8-3.1)**

- ☐ Within each lesson, special education students are given choice of topic and resources so that their materials are within their ability level and high-interest.
- ☐ All content will be modeled with examples and all essays are built on a step-by-step basis so modifications for assignments in small chunks are met.

All other IEP modifications will be honored (ie. hard copies of notes, directions restated, etc.)

## **Interdisciplinary Connections**

---

**ELA** - Students will apply reasoning skills to justify statements. Students will justify statements through oral

and written communication.

**SCIENCE** - Students will analyze the distance, average velocity, and instantaneous velocity of objects.

**SOCIAL STUDIES** - Brief historical notes are found throughout both textbooks. They present stories of people and the research that they have done to advance the study of mathematics.

**WORLD LANGUAGES** -

**VISUAL/PERFORMING ARTS** -

**APPLIED TECHNOLOGY** -

**BUSINESS EDUCATION** -

**GLOBAL AWARENESS** -

# Unit 2: Graphs of Trigonometric Functions

Content Area: **Math**  
Course(s): **MATH ANALYSIS**  
Time Period: **Marking Period 1**  
Length: **4 weeks**  
Status: **Published**

## Standards

---

### Math Standards

---

MA.F-TF.A.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
MA.F-TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
MA.F-TF.B.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

### Mathematical Practices

---

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.

### Life Literacies and Key Skills

---

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.CT.4	Participate in online strategy and planning sessions for course-based, school-based, or other project and determine the strategies that contribute to effective outcomes.
TECH.9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).



## Transfer Goals

---

## Transfer Goals

---

Students will be able to independently use their learning to understand that graphs of Trigonometric Functions are directly related to the angles of the unit circle.

## Concepts

---

## Essential Questions

---

- How do transformations affect the trigonometric graphs of each function?
- How do you determine the period and amplitude of a trigonometric function without looking at the graph of the function?
- How do you graph the basic trigonometric functions on the coordinate plane?
- How do you use graphs or trigonometric functions to determine trigonometric identities?
- What are trigonometric functions used to model in real life?

## Understandings

---

- Trigonometric Functions are used to model harmonic and oscillating behavior.

## Critical Knowledge and Skills

---

## Knowledge

---

Students will know:

- Graphs of Trigonometric Functions: Sine, Cosine, Cosecant, Secant, Tangent, and Cotangent
- Amplitude
- Period

- Phase Shift
- Graphs of Inverse Trigonometric Functions: Arcsine, Arccosine, and Arctangent

## **Skills**

---

Students will be able to:

- Sketch graphs of trigonometric functions.
- Identify the length of period, amplitude and domain and range of trigonometric functions.

## **Assessment and Resources**

---

### **School Formative Assessment Plan (Other Evidence)**

---

- Homework
- Quizzes
- Exit Tickets
- Reflections
- Performance Tasks

- Picture Walk to identify the trigonometric graph. Identify if the equation matches the graph.

-Trigonomemtric Art Project: This project consists of creating a picture or design by graphing at least 10 trigonometric functions on top of each other using the parameters of of the given grid.

### **School Summative Assessment Plan**

---

- Unit Assessment

## **Primary Resources**

---

- Larsen, Ron, Hostetler, Robert P. and Edwards. Bruce H. *Pre-Calculus with Limits: A Graphing Approach* Houghton Mifflin Company, 2005

## **Supplementary Resources**

---

- IXL
- Kutasoftware
- Desmos
- Khan Academy
- PatrickJMT

## **Technology Integration and Differentiated Instruction**

---

### **Technology Integration**

---

- **Google Products**

- Google Classroom - Used for daily interactions with the students covering a vast majority of different educational resources (Daily Notes, Exit Tickets, Classroom Polls, Quick Checks, Additional Resources/ Support, Homework, etc.)
- GAFE (Google Apps For Education) - Using various programs connected with Google to collaborate within the district, co-teachers, grade level partner teacher, and with students to stay connected with the content that is covered within the topic. Used to collect data in real time and see results upon completion of the assignments to allow for 21st century learning.

- **One to One Student's laptop**

- All students within the West Deptford School District are given a computer, allowing for 21st century learning to occur within every lesson/topic.

- **Additional Support Videos**

- Videos can be assigned from PatrickJMT, Khan Academy and Youtube, etc... to support each of the lessons within this topic.

## **Differentiated Instruction**

---

### **Gifted Students (N.J.A.C.6A:8-3.1)**

- ☐ Within each lesson, the Gifted Students are given choice on topic and subject matter allowing them to explore interests appropriate to their abilities, areas of interest and other courses.

### **English Language Learners (N.J.A.C.6A:15)**

- ☐ Within each lesson, the English Language Learners are given choice of topic and resources so that their materials are within their ability to grasp the language.
- ☐ Work with ELL Teacher to allow for all assignments to be completed with extra time.

### **At-Risk Students (N.J.A.C.6A:8-4.3c)**

- ☐ Within each lesson, the at-risk students are given choice of topic and resources so that their materials are within their ability level and high-interest.

### **Special Education Students (N.J.A.C.6A:8-3.1)**

- ☐ Within each lesson, special education students are given choice of topic and resources so that their materials are within their ability level and high-interest.
- ☐ All content will be modeled with examples and all essays are built on a step-by-step basis so modifications for assignments in small chunks are met.

All other IEP modifications will be honored (ie. hard copies of notes, directions restated, etc.)

## **Interdisciplinary Connections**

---

**ELA** - Students will apply reasoning skills to justify statements. Students will justify statements through oral and written communication.

**SCIENCE** - Students will analyze the distance, average velocity, and instantaneous velocity of objects.

**SOCIAL STUDIES** - Brief historical notes are found throughout both textbooks. They present stories of people and the research that they have done to advance the study of mathematics.

**WORLD LANGUAGES** -

**VISUAL/PERFORMING ARTS** -

**APPLIED TECHNOLOGY** -

**BUSINESS EDUCATION** -

**GLOBAL AWARENESS** -

### **Learning Plan / Pacing Guide**

---

Suggested timeline: **Honors will work at an accelerated pace. They will complete this unit in three weeks.**

- Week 1
  - Review of factoring and fractions
  - Review of basic graphs
  - Activity - Student will construct graphs of sine and cosine using a unit circle that they constructed. Students will then using string and rules to construct the graphs.
- Week 2
  - Sketching secant/cosecant/tangent/cotangent functions
  - Quiz
  - Project - Students will complete The Trigonometric Art Project. Students will either create a design or use a picture to find equations of trigonometric functions in the design/picture.
- Week 3
  - Writing functions given trig graphs
  - Quiz
- Week 4
  - Inverse trig functions
  - Unit Test

# Unit 3: Analytic Trigonometry

Content Area:	Math
Course(s):	MATH ANALYSIS
Time Period:	Marking Period 2
Length:	4/5 weeks
Status:	Published

## Standards

---

### Math Standards

---

MA.F-TF.C	Prove and apply trigonometric identities
MA.F-TF.C.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$ , $\cos(\theta)$ , or $\tan(\theta)$ given $\sin(\theta)$ , $\cos(\theta)$ , or $\tan(\theta)$ and the quadrant of the angle.
MA.F-TF.C.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

### Mathematical Practices

---

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.

### Life Literacies and Key Skills

---

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

### Transfer Goals and Career Ready Practices

---

## **Transfer Goals**

---

- Use standard algebraic techniques and inverse trigonometric functions to solve trigonometric equations.
- Use sum and difference formulas, multiple-angle formulas, half-angle formulas, and product-to-sum formulas to rewrite and evaluate trigonometric functions.
- Use the fundamental trigonometric identities to evaluate trigonometric functions and simplify trigonometric expressions.
- Verify trigonometric identities.

## **Concepts**

---

## **Essential Questions**

---

- What is the purpose of the trigonometric identities, sum and difference formulas, multiple angle formulas, and product-to-sum formulas?
- How do you graphically solve a trigonometric equation?
- How is proving or verifying a trigonometric identity different then solving a trigonometric equation?
- What are the relationships between the Pythagorean Identities for Trigonometry?

## **Understandings**

---

- Trigonometric identities are used to simplify and evaluate trigonometric functions and expressions.
- Sum and difference formulas, multiple-angle formulas, and product-to-sum formulas are used to rewrite and evaluate trigonometric functions.

## **Critical Knowledge and Skills**

---

## **Knowledge**

---

Students will know:

- Trigonometric Identities: Reciprocal, Quotient, Pythagorean, Cofunction, Even/Odd
- Sum and Difference Formulas
- Multiple-Angle Formulas
- Half-Angle Formulas
- Product-to-Sum Formulas

## **Skills**

---

Students will be able to:

- Use the fundamental trig identities to evaluate trig functions and simplify trigonometric expressions.
- Verify trig identities.
- Use standard algebraic techniques and inverse trigonometric functions to solve trigonometric equations.
- Use sum and difference formulas, multiple-angle formulas, half-angle formulas, and product-to-sum formulas to rewrite and evaluate trigonometric functions.

## **Assessment and Resources**

---

### **School Formative Assessment Plan (Other Evidence)**

---

- Homework
- Quizzes
- Exit Tickets
- Reflections
- Performance Tasks

### **School Summative Assessment Plan**

---

- Unit Assessment



## **Primary Resources**

---

- Larsen, Ron, Hostetler, Robert P. and Edwards. Bruce H. *Pre-Calculus with Limits: A Graphing Approach* Houghton Mifflin Company, 2005

## **Supplementary Resources**

---

- IXL
- Kutasoftware
- Desmos
- Khan Academy
- PatrickJMT

## **Technology Integration and Differentiated Instruction**

---

### **Technology Integration**

---

#### **• Google Products**

- Google Classroom - Used for daily interactions with the students covering a vast majority of different educational resources (Daily Notes, Exit Tickets, Classroom Polls, Quick Checks, Additional Resources/ Support, Homework, etc.)
- GAFE (Google Apps For Education) - Using various programs connected with Google to collaborate within the district, co-teachers, grade level partner teacher, and with students to stay connected with the content that is covered within the topic. Used to collect data in real time and see results upon completion of the assignments to allow for 21st century learning.

#### **• One to One Student's laptop**

- All students within the West Deptford School District are given a computer, allowing for 21st century learning to occur within every lesson/topic.

#### **• Additional Support Videos**

- Videos can be assigned from PatrickJMT, Khan Academy and Youtube, etc... to support each of the lessons within this topic.

## **Differentiated Instruction**

---

### **Gifted Students (N.J.A.C.6A:8-3.1)**

- ☐ Within each lesson, the Gifted Students are given choice on topic and subject matter allowing them to explore interests appropriate to their abilities, areas of interest and other courses.

### **English Language Learners (N.J.A.C.6A:15)**

- ☐ Within each lesson, the English Language Learners are given choice of topic and resources so that their materials are within their ability to grasp the language.
- ☐ Work with ELL Teacher to allow for all assignments to be completed with extra time.

### **At-Risk Students (N.J.A.C.6A:8-4.3c)**

- ☐ Within each lesson, the at-risk students are given choice of topic and resources so that their materials are within their ability level and high-interest.

### **Special Education Students (N.J.A.C.6A:8-3.1)**

- ☐ Within each lesson, special education students are given choice of topic and resources so that their materials are within their ability level and high-interest.
- ☐ All content will be modeled with examples and all essays are built on a step-by-step basis so modifications for assignments in small chunks are met.

All other IEP modifications will be honored (ie. hard copies of notes, directions restated, etc.)

## **Interdisciplinary Connections**

---

**ELA** - Students will apply reasoning skills to justify statements. Students will justify statements through oral and written communication.

**SCIENCE** - Students will analyze the distance, average velocity, and instantaneous velocity of objects.

**SOCIAL STUDIES** - Brief historical notes are found throughout both textbooks. They present stories of

people and the research that they have done to advance the study of mathematics.

**WORLD LANGUAGES -**

**VISUAL/PERFORMING ARTS -**

**APPLIED TECHNOLOGY -**

**BUSINESS EDUCATION -**

**GLOBAL AWARENESS -**

---

### **Learning Plan / Pacing Guide**

Timeline - **Honors will work at an accelerated rate. Honors will complete this unit in four weeks.**

Week 1

- Simplify Trigonometric Expressions
- Quiz
- Activity - Silent Journal - Students will work in pairs for this activity. Students will work on the same problem and then share their answers with their partner. Students must critique work without using their voices. Students must written critique to "talk" to their partner.

Week 2

- Verify Trigonometric Identities
- Test
- Activity - Poster Paper - They will be trigonometric identities writing on paper throughout the room. Students will have markers and will work around the room and write only one line of the solution of the problem. Students will continue working on problems unit all are complete and correct.

Week 3

- Solving Trigonometric Equations
- Quiz

Week 4

- Sum and Difference Formulas
- Double Angle Formulas
- Power Reducing Formulas
- Half Angle Formulas
- Product to Sum Formulas
- Sum to Product Formulas
- Quiz

Week 5

- Review and Test

# Unit 4: Law of Sines and Law of Cosines

Content Area: **Math**  
Course(s): **MATH ANALYSIS**  
Time Period: **Marking Period 2**  
Length: **3 weeks**  
Status: **Published**

## Standards

---

### Math Standards

---

MA.G-SRT.D	Apply trigonometry to general triangles
MA.G-SRT.D.9	Derive the formula $A = (1/2)ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
MA.G-SRT.D.10	Prove the Laws of Sines and Cosines and use them to solve problems.
MA.G-SRT.D.11	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

### Mathematical Practices

---

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.

### Life Literacies and Key Skills

---

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12prof.CR3.a).

### Transfer Goals

---

## **Transfer Goals**

---

- Find the area of oblique triangles.
- Use the Law of Sines and the Law of Cosines to model and solve real-life problems.
- Use the Law of Sines and the Law of Cosines to solve oblique triangles.

## **Concepts**

---

## **Essential Questions**

---

- What is the relationship of the Law of Sines, Law of Cosines and area formula to theorems you previously learned?
- What is the utility of the Law of Sines and the Law of Cosines?
- When is it necessary to use the Law of Sines to solve a triangle?
- Why do we use the Law of Sine and Law of Cosines?

## **Understandings**

---

- The Law of Sines and Law of Cosines are used to solve oblique triangles.

## **Critical Knowledge and Skills**

---

## **Knowledge**

---

Students will know:

- Law of Sines and Law of Cosines
- Oblique Triangle
- Heron's Area Formula

## **Skills**

---

Students will be able to:

- Use the Law of Sines and the Law of Cosines to solve oblique triangles.
- Find the areas of oblique triangles.
- Use Law of Sines and Cosines to model and solve real-life problems.

## **Assessment and Resources**

---

### **School Formative Assessment Plan (Other Evidence)**

---

- Homework
- Quizzes
- Exit Tickets
- Reflections
- Performance Tasks
  - Law of Sines Activity
  - Roller Coaster Activity

### **School Summative Assessment Plan**

---

- Unit Assessment

## **Primary Resources**

---

- Larsen, Ron, Hostetler, Robert P. and Edwards. Bruce H. *Pre-Calculus with Limits: A Graphing Approach* Houghton Mifflin Company, 2005

## **Supplementary Resources**

---

- IXL
- Kutasoftware
- Desmos
- Khan Academy
- PatrickJMT

## **Technology Integration and Differentiated Instruction**

---

### **Technology Integration**

---

#### **• Google Products**

- Google Classroom - Used for daily interactions with the students covering a vast majority of different educational resources (Daily Notes, Exit Tickets, Classroom Polls, Quick Checks, Additional Resources/ Support, Homework, etc.)
- GAFE (Google Apps For Education) - Using various programs connected with Google to collaborate within the district, co-teachers, grade level partner teacher, and with students to stay connected with the content that is covered within the topic. Used to collect data in real time and see results upon completion of the assignments to allow for 21st century learning.

#### **• One to One Student's laptop**

- All students within the West Deptford School District are given a computer, allowing for 21st century learning to occur within every lesson/topic.

#### **• Additional Support Videos**

- Videos can be assigned from PatrickJMT, Khan Academy and Youtube, etc... to support each of the lessons within this topic.

## **Differentiated Instruction**

---



### **Gifted Students (N.J.A.C.6A:8-3.1)**

☐ Within each lesson, the Gifted Students are given choice on topic and subject matter allowing them to explore interests appropriate to their abilities, areas of interest and other courses.

### **English Language Learners (N.J.A.C.6A:15)**

☐ Within each lesson, the English Language Learners are given choice of topic and resources so that their materials are within their ability to grasp the language.

☐ Work with ELL Teacher to allow for all assignments to be completed with extra time.

### **At-Risk Students (N.J.A.C.6A:8-4.3c)**

☐ Within each lesson, the at-risk students are given choice of topic and resources so that their materials are within their ability level and high-interest.

### **Special Education Students (N.J.A.C.6A:8-3.1)**

☐ Within each lesson, special education students are given choice of topic and resources so that their materials are within their ability level and high-interest.

☐ All content will be modeled with examples and all essays are built on a step-by-step basis so modifications for assignments in small chunks are met.

All other IEP modifications will be honored (ie. hard copies of notes, directions restated, etc.)

### **Interdisciplinary Connections**

---

**ELA** - Students will apply reasoning skills to justify statements. Students will justify statements through oral and written communication.

**SCIENCE** - Students will analyze the distance, average velocity, and instantaneous velocity of objects.

**SOCIAL STUDIES** - Brief historical notes are found throughout both textbooks. They present stories of people and the research that they have done to advance the study of mathematics.

**WORLD LANGUAGES** -

**VISUAL/PERFORMING ARTS** -

**APPLIED TECHNOLOGY** -

**BUSINESS EDUCATION -**

**GLOBAL AWARENESS -**

### **Learning Plan / Pacing Guide**

---

Timeline - **Timeline is the same for honors and college prep due to additional topic of bearing.**

#### **Week 1**

- Law of Sines - Ambiguous case
- Law of Sines - Word problems/Applications
- Bearing - Word problems/Applications - Honors Topic
- Quiz

#### **Week 2**

- Law of Cosines - Solving triangles
- Law of Cosines - Word problems/Applications
- Quiz

#### **Week 3**

- Area formulas
- Review
- Test

# Unit 5: Vectors

Content Area: **Math**  
Course(s): **MATH ANALYSIS**  
Time Period: **Marking Period 2**  
Length: **3 weeks**  
Status: **Published**

## Standards

---

### Math Standards

---

MA.N-VM.A.1	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\vec{v}$ , $ \vec{v} $ , $  \vec{v}  $ , $v$ ).
MA.N-VM.A.2	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
MA.N-VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.
MA.N-VM.B.4a	Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
MA.N-VM.B.4b	Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
MA.N-VM.B.4c	Understand vector subtraction $\vec{u} - \vec{v}$ as $\vec{u} + (-\vec{v})$ , where $-\vec{v}$ is the additive inverse of $\vec{v}$ , with the same magnitude as $\vec{v}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
MA.N-VM.B.5a	Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $3(\vec{u}_x, \vec{u}_{\text{subscript } y}) = (3\vec{u}_x, 3\vec{u}_{\text{subscript } y})$ .
MA.N-VM.B.5b	Compute the magnitude of a scalar multiple $\vec{c}\vec{u}$ using $  \vec{c}\vec{u}   =  \vec{c}  \vec{u} $ . Compute the direction of $\vec{c}\vec{u}$ knowing that when $ \vec{c}  \vec{u}  \neq 0$ , the direction of $\vec{c}\vec{u}$ is either along $\vec{u}$ (for $\vec{c} > 0$ ) or against $\vec{u}$ (for $\vec{c} < 0$ ).

### Mathematical Practices

---

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.

## Life Literacies and Key Skills

---

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

## Transfer Goals

---

### Transfer Goals

---

Students will be able to independently use their learning to...

• Find the direction angles of vectors.	.
• Find the dot product of two vectors and use properties of the dot product. (Honors Only)	.
• Perform basic vector operations and represent vectors graphically.	.
• Represent vectors as directed line segments.	.
• Use vectors to model and solve real-life problems.	.
• Write the component form of vectors.	.
• Write vectors as linear combinations of unit vectors.	.
• Write vectors as sums of two vector components. (Honors Only)	.

## Concepts

---

### Essential Questions

---

- How do you find the dot product of two vectors?
- How do you manipulate vector with Scalars?
- How do you multiply vectors with other vectors
- What is a vector?

- What is the difference between vectors and rays?

## **Understandings**

---

- Vectors are used to model quantities that involve both magnitude and direction.

## **Critical Knowledge and Skills**

---

## **Knowledge**

---

*Students will know...*

- Vectors
- Component Form
- Magnitude
- Direction Angle
- Scalar Multiplication
- Vector Addition
- Dot Product\*

## **Skills**

---

*Students will be able to...*

- Represent vectors as directed line segments.
- Write the component form of vectors.
- Perform basic vector operations and represent vectors graphically.
- Write vectors as linear combinations of unit vectors.
- Find the direction angles of vectors.
- Find the dot product of two vectors and use properties of the dot product.\*
- Write vectors as sums of two vector components.\*
- Use vectors to model and solve real-life problems.

\*Topics for Honors Math Analysis Only

## **Assessment and Resources**

---

### **School Formative Assessment Plan (Other Evidence)**

---

- Homework
- Quizzes
- Exit Tickets
- Reflections
- Performance Tasks

### **School Summative Assessment Plan**

---

- Unit Assessment

### **Primary Resources**

---

- Larson, Ron. Precalculus with Limits - A Graphing Approach. Houghton Mifflin Company, 2005

### **Supplementary Resources**

---

- IXL
- Keta Software
- Desmos
- Khan Academy

## **Technology Integration and Differentiated Instruction**

---

## **Technology Integration**

---

### **● Google Products**

- Google Classroom - Used for daily interactions with the students covering a vast majority of different educational resources (Daily Notes, Exit Tickets, Classroom Polls, Quick Checks, Additional Resources/ Support, Homework, etc.)
- GAFE (Google Apps For Education) - Using various programs connected with Google to collaborate within the district, co-teachers, grade level partner teacher, and with students to stay connected with the content that is covered within the topic. Used to collect data in real time and see results upon completion of the assignments to allow for 21st century learning.

### **● One to One Student's laptop**

- All students within the West Deptford School District are given a computer, allowing for 21st century learning to occur within every lesson/topic.

### **● Additional Support Videos**

Videos can be assigned from PatrickJMT, Khan Academy, YouTube, etc, to support each of the lessons within this topic.

## **Differentiated Instruction**

---

### **Gifted Students (N.J.A.C.6A:8-3.1)**

- ☐ Within each lesson, the Gifted Students are given choice on topic and subject matter allowing them to explore interests appropriate to their abilities, areas of interest and other courses.

### **English Language Learners (N.J.A.C.6A:15)**

- ☐ Within each lesson, the English Language Learners are given choice of topic and resources so that their materials are within their ability to grasp the language.
- ☐ Work with ELL Teacher to allow for all assignments to be completed with extra time.

### **At-Risk Students (N.J.A.C.6A:8-4.3c)**

☐ Within each lesson, the at-risk students are given choice of topic and resources so that their materials are within their ability level and high-interest.

### **Special Education Students (N.J.A.C.6A:8-3.1)**

☐ Within each lesson, special education students are given choice of topic and resources so that their materials are within their ability level and high-interest.

☐ All content will be modeled with examples and all essays are built on a step-by-step basis so modifications for assignments in small chunks are met.

All other IEP modifications will be honored (ie. hard copies of notes, directions restated, etc.)

### **Interdisciplinary Connections**

---

**ELA** - Students will apply reasoning skills to justify statements. Students will justify statements through oral and written communication.

**SCIENCE** - Students will analyze the distance, average velocity, and instantaneous velocity of objects.

**SOCIAL STUDIES** - Brief historical notes are found throughout both textbooks. They present stories of people and the research that they have done to advance the study of mathematics.

**WORLD LANGUAGES** -

**VISUAL/PERFORMING ARTS** -

**APPLIED TECHNOLOGY** -

**BUSINESS EDUCATION** -

**GLOBAL AWARENESS** -

### **Learning Plan / Pacing Guide**

---

Timeline - Honors will be 4 weeks. CP will be 3 weeks.

Week 1

- Component form of a vector
- Vector addition and scalar multiplication



- Unit Vectors
- Sketching vectors
- Quiz

## Week 2

- Linear combination of unit vectors
- Direction angles
- Application problems

## Week 3

- Application problems
- Review
- Test

## Week 4 - Honors only

- Vectors and Dot Products
- Angles between vectors and determine whether two vectors are orthogonal
- Write vectors as sums of two vector components
- Quiz

• Vectors Treasure Hunt	.
-------------------------	---

-

# Unit 6: Linear Systems and Matrices

Content Area: **Math**  
Course(s): **Generic Course**  
Time Period: **Marking Period 3**  
Length: **4 weeks**  
Status: **Published**

## Standards

---

### Math Standards

---

MA.N-VM.C.6	Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
MA.N-VM.C.7	Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
MA.N-VM.C.8	Add, subtract, and multiply matrices of appropriate dimensions.
MA.N-VM.C.9	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
MA.N-VM.C.10	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
MA.N-VM.C.11	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
MA.N-VM.C.12	Work with $2 \times 2$ matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.
MA.A-REI.C.8	Represent a system of linear equations as a single matrix equation in a vector variable.
MA.A-REI.C.9	Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension $3 \times 3$ or greater).

### Mathematical Practices

---

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.

## Life Literacies and Key Skills

---

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
TECH.9.4.12.TL.2	Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

## Transfer Goals

---

## Transfer Goals

---

Students will be able to independently use their learning to understand how operations with matrices can be used to solve real-life problems

## Concepts

---

## Essential Questions

---

- How can you find the inverse and determinant of a matrix?
- How can you use matrices to perform geometrical transformations?
- How can you use matrices to represent systems of equations?
- How do you add, subtract, and multiply matrices?
- What is the purpose of learning about matrices?

## Understandings

---

Students will understand that . . .

• • Matrices can be used to solve systems of linear equations in two or more variables.	.
---	---

---

## Critical Knowledge and Skills

---

### Knowledge

*Students will know...*

- Multivariable Linear System
- Matrix: Augmented and Coefficient
- Row-Echelon Form
- Elementary Row Operations
- Matrix Addition, Scalar Multiplication, and Matrix Multiplication
- Identity Matrix
- Square Matrix
- Inverse
- Determinant

---

### Skills

Students will be able to:

- Use matrices, back-substitution and elimination to solve systems of linear equations.
- Write matrices, identify their orders, and perform elementary row operations on matrices.
- Add, subtract, and multiply matrices.
- Find inverse of matrices.
- Find determinants of square matrices.
- Use inverse matrices to solve systems of linear equations.
- Use determinants to find areas of triangles.

Use systems of equations and matrices to model and solve real-life problems. HMA only

## **Assessment and Resources**

---

### **School Formative Assessment Plan (Other Evidence)**

---

- Homework
- Quizzes
- Exit Tickets
- Reflections
- Performance Tasks

### **School Summative Assessment Plan**

---

- Unit Assessment

### **Primary Resources**

---

- Larson, Ron. Precalculus with Limits - A Graphing Approach. Houghton Mifflin Company, 2005

### **Supplementary Resources**

---

- IXL
- Kutasoftware
- Desmos
- Khan Academy
- PatrickJMP

YouTube

## **Technology Integration and Differentiated Instruction**

---

## **Technology Integration**

---

### **● Google Products**

- Google Classroom - Used for daily interactions with the students covering a vast majority of different educational resources (Daily Notes, Exit Tickets, Classroom Polls, Quick Checks, Additional Resources/ Support, Homework, etc.)
- GAFE (Google Apps For Education) - Using various programs connected with Google to collaborate within the district, co-teachers, grade level partner teacher, and with students to stay connected with the content that is covered within the topic. Used to collect data in real time and see results upon completion of the assignments to allow for 21st century learning.

### **● One to One Student's laptop**

- All students within the West Deptford School District are given a computer, allowing for 21st century learning to occur within every lesson/topic.

### **● Additional Support Videos**

Videos can be assigned from PatrickJMT, Khan Academy, YouTube, etc, to support each of the lessons within this topic.

## **Differentiated Instruction**

---

### **Gifted Students (N.J.A.C.6A:8-3.1)**

- ☐ Within each lesson, the Gifted Students are given choice on topic and subject matter allowing them to explore interests appropriate to their abilities, areas of interest and other courses.

### **English Language Learners (N.J.A.C.6A:15)**

- ☐ Within each lesson, the English Language Learners are given choice of topic and resources so that their materials are within their ability to grasp the language.

- ☐ Work with ELL Teacher to allow for all assignments to be completed with extra time.

### **At-Risk Students (N.J.A.C.6A:8-4.3c)**

- ☐ Within each lesson, the at-risk students are given choice of topic and resources so that their materials are within their ability level and high-interest.

### **Special Education Students (N.J.A.C.6A:8-3.1)**

- ☐ Within each lesson, special education students are given choice of topic and resources so that their materials are within their ability level and high-interest.
- ☐ All content will be modeled with examples and all essays are built on a step-by-step basis so modifications for assignments in small chunks are met.

All other IEP modifications will be honored (ie. hard copies of notes, directions restated, etc.)

### **Interdisciplinary Connections**

---

**ELA** - Students will apply reasoning skills to justify statements. Students will justify statements through oral and written communication.

**SCIENCE** - Students will analyze the distance, average velocity, and instantaneous velocity of objects.

**SOCIAL STUDIES** - Brief historical notes are found throughout both textbooks. They present stories of people and the research that they have done to advance the study of mathematics.

**WORLD LANGUAGES** -

**VISUAL/PERFORMING ARTS** -

**APPLIED TECHNOLOGY** -

**BUSINESS EDUCATION** -

**GLOBAL AWARENESS** -

### **Learning Plan / Pacing Guide**

---

This is a mainly self-paced unit. Students will be given a list of topics to be covered with a guided notes packet. Students will work on packet and take the quizzes after each section when they are ready. The first three weeks are used to cover the matrices material at the students own pace with a test given at the end of the third week. There is a project due at the end of the unit. Students will create an encoding matrix and encode

a message to be sent to a partner. The last week is teacher led instruction covering partial fraction decomposition.



# Unit 7: Analytic Geometry

Content Area: **Math**  
Course(s): **Generic Course, MATH ANALYSIS**  
Time Period: **Marking Period 4**  
Length: **4 weeks**  
Status: **Published**

## Standards

---

### Math Standards

---

MA.F-IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MA.G-CO.A.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
MA.G-GPE.A.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
MA.G-GPE.A.2	Derive the equation of a parabola given a focus and directrix.
MA.G-GPE.A.3	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

### Mathematical Practices

---

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.

### Life Literacies and Key Skills

---

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

## Transfer Goals

---

## Transfer Goals

---

Students will be able to independently use their learning to...

• Analyze and sketch parabolas, ellipses, and hyperbolas.	.
• Evaluate sets of parametric equations for given values of the parametric and graph curves that are represented by sets of parametric equations.	.
• Graph polar equations.	.
• Plot points in the polar coordinate system and convert equations from rectangular to polar and vice versa.	.
• Rewrite sets of parametric equations as single rectangular equations and find sets of parametric equations for graphs.	.
• Write conics in terms of eccentricity and write equations on conics in polar form.	.
• Write the standard forms of the equations of basic conics.	.

## Concepts

---

## Essential Questions

---

- How do we write equations of ellipses with only the characteristics of the ellipse?
- How do we determine what the conic represents?
- How do we know the length and width of an ellipse?
- How do you graph curves using the Polar Coordinate system?\*
- How does the concept of distance relate to the concepts of ellipses and hyperbolas?
- What are conic sections?
- What are parametric equations used for?
- What is the Polar Coordinate system?\*
- What is the importance of conic sections?
- Why is the polar coordinate system studied?

## Understandings

---

*Students will understand that...*

- Conic sections are curves that result from the intersection of a plane and a cone.
- Parametric equations use a third variable to represent a curve in a plane.
- Polar coordinates offer a different mathematical perspective on graphing.

## **Critical Knowledge and Skills**

---

### **Knowledge**

---

*Students will know...*

- Ellipse
- Hyperbolas
- Parametric Equations
- Polar Coordinate System
- Eccentricity \*HMA only

### **Skills**

---

Students will be able to:

- Write the standard forms of the equations of basic conics.
- Analyze and sketch parabolas, ellipses, and hyperbolas.
- Evaluate sets of parametric equations for given values of the parametric and graph curves that are represented by sets of parametric equations.
- Rewrite sets of parametric equations as single rectangular equations and find sets of parametric equations for graphs.
- Plot points in the polar coordinate system and convert equations from rectangular to polar form and vice versa.
- Graph polar equations.
- Write conics in terms of eccentricity and write Conic equations in polar form.

## **Assessment and Resources**

---

### **School Formative Assessment Plan (Other Evidence)**

---

- Homework
- Quizzes
- Exit Tickets
- Reflections
- Performance Tasks

### **School Summative Assessment Plan**

---

- Unit Assessment

### **Primary Resources**

---

- Larson, Ron. Precalculus with Limits - A Graphing Approach. Houghton Mifflin Company, 2005

### **Supplementary Resources**

---

- IXL
- Kutasoftware
- Desmos
- Khan Academy
- PatrickJMP

YouTube

## **Technology Integration and Differentiated Instruction**

---

## **Technology Integration**

---

### **● Google Products**

- Google Classroom - Used for daily interactions with the students covering a vast majority of different educational resources (Daily Notes, Exit Tickets, Classroom Polls, Quick Checks, Additional Resources/ Support, Homework, etc.)
- GAFE (Google Apps For Education) - Using various programs connected with Google to collaborate within the district, co-teachers, grade level partner teacher, and with students to stay connected with the content that is covered within the topic. Used to collect data in real time and see results upon completion of the assignments to allow for 21st century learning.

### **● One to One Student's laptop**

- All students within the West Deptford School District are given a computer, allowing for 21st century learning to occur within every lesson/topic.

### **● Additional Support Videos**

Videos can be assigned from PatrickJMT, Khan Academy, YouTube, etc, to support each of the lessons within this topic.

## **Differentiated Instruction**

---

### **Gifted Students (N.J.A.C.6A:8-3.1)**

- ☐ Within each lesson, the Gifted Students are given choice on topic and subject matter allowing them to explore interests appropriate to their abilities, areas of interest and other courses.

### **English Language Learners (N.J.A.C.6A:15)**

- ☐ Within each lesson, the English Language Learners are given choice of topic and resources so that their materials are within their ability to grasp the language.

- ☐ Work with ELL Teacher to allow for all assignments to be completed with extra time.

### **At-Risk Students (N.J.A.C.6A:8-4.3c)**

- ☐ Within each lesson, the at-risk students are given choice of topic and resources so that their materials

are within their ability level and high-interest.

### **Special Education Students (N.J.A.C.6A:8-3.1)**

- ☐ Within each lesson, special education students are given choice of topic and resources so that their materials are within their ability level and high-interest.
- ☐ All content will be modeled with examples and all essays are built on a step-by-step basis so modifications for assignments in small chunks are met.

All other IEP modifications will be honored (ie. hard copies of notes, directions restated, etc.)

### **Interdisciplinary Connections**

---

**ELA** - Students will apply reasoning skills to justify statements. Students will justify statements through oral and written communication.

**SCIENCE** - Students will analyze the distance, average velocity, and instantaneous velocity of objects.

**SOCIAL STUDIES** - Brief historical notes are found throughout both textbooks. They present stories of people and the research that they have done to advance the study of mathematics.

**WORLD LANGUAGES** -

**VISUAL/PERFORMING ARTS** -

**APPLIED TECHNOLOGY** -

**BUSINESS EDUCATION** -

**GLOBAL AWARENESS** -

### **Learning Plan / Pacing Guide**

---

Timeline - Honors and CP complete this unit in the same amount of time.

Week 1

- Parabolas
- Circles
- Ellipses
- Activity - Students will create an ellipse using string of fixed length. Students will label all important parts of the ellipse

- Quiz

## Week 2

- Hyperbolas
- Review
- Test

## Week 3

- Parametric Equations
- Quiz

## Week 4

- Polar Coordinates
- Converting from rectangular/polar and polar/rectangular coordinates
- Converting from rectangular/polar and polar/rectangular equations
- Quiz

• Parabola Characteristics	.
• Tic Tac Toe - Conic Sections	.

-

# Unit 8: Limits

Content Area:	<b>Math</b>
Course(s):	<b>Generic Course, MATH ANALYSIS, CALCULUS HONORS, AP CALC</b>
Time Period:	<b>Marking Period 4</b>
Length:	<b>5 Weeks</b>
Status:	<b>Published</b>

## Standards

---

### Math Standards

---

MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.F-LE.A.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
MA.A-APR.D.6	Rewrite simple rational expressions in different forms; write $\frac{p(x)}{q(x)}$ in the form $\frac{p(x)}{q(x)} + \frac{r(x)}{q(x)}$ , where $p(x)$ , $q(x)$ , $r(x)$ , and $s(x)$ are polynomials with the degree of $p(x)$ less than the degree of $q(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.

### Mathematical Practices

---

- 1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression.
- 1.E Apply appropriate mathematical rules or procedures, with and without technology.
- 2.B Identify mathematical information from graphical, symbolic, numerical, and/or verbal representations.
- 2.C Identify a re-expression of mathematical information presented in a given representation.
- 2.D Identify how mathematical characteristics or properties of functions are related in different representations.
- 3.B Identify an appropriate mathematical definition, theorem, or test to apply.
- 3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.
- 3.D Apply an appropriate mathematical definition, theorem, or test.
- 3.E Provide reasons or rationales for solutions or conclusions.

### Life Literacies and Key Skills

---

TECH.9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
------------------	---



TECH.9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3).
TECH.9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).

## Transfer Goals

---

## Transfer Goals

---

Limits introduce the subtle distinction between evaluating a function at a point and considering what value the function is approaching, if any, as  $x$  approaches a point.

## Concepts

---

## Essential Questions

---

- Can change occur at an instant?
- How does knowing the value of a limit, or that a limit does not exist, help you to make sense of interesting features of functions and their graphs?
- How do we close loopholes so that a conclusion about a function is always true?
- How do we know if a limit exists?
- How do you evaluate a limit algebraically?
- How do you evaluate a limit using a graph?

## Understandings

---

- Calculus allows us to generalize knowledge about motion to diverse problems involving change.
- Reasoning with definitions, theorems, and properties can be used to justify claims about limits.
- Reasoning with definitions, theorems, and properties can be used to justify claims about continuity.
- Existence theorems allow us to draw conclusions about a function's behavior on an interval without precisely locating that behavior.

## Critical Knowledge and Skills

---

## Knowledge

---

Students will know:

- Calculus uses limits to understand and model dynamic change.
- Because an average rate of change divides the change in one variable by the change in another, the average rate of change is undefined at a point where the change in the independent variable would be zero.
- The limit concept allows us to define instantaneous rate of change in terms of average rates of change.
- Given a function  $f$ , the limit of  $f(x)$  as  $x$  approaches  $c$  is a real number  $R$  if  $f(x)$  can be made arbitrarily close to  $R$  by taking  $x$  sufficiently close to  $c$  (but not equal to  $c$ ). If the limit exists and is a real number, then the common notation is  $\lim f(x) = R$ .
- A limit can be expressed in multiple ways, including graphically, numerically, and analytically.
- The concept of a limit includes one sided limits.
- Graphical information about a function can be used to estimate limits.
- Because of issues of scale, graphical representations of functions may miss important function behavior.
- A limit might not exist for some functions at particular values of  $x$ . Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.
- Numerical information can be used to estimate limits.
- One-sided limits can be determined analytically or graphically.
- Limits of sums, differences, products, quotients, and composite functions can be found using limit theorems.
- It may be necessary or helpful to rearrange expressions into equivalent forms before evaluating limits.
- The limit of a function may be found by using the squeeze theorem.
- Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.
- A function  $f$  is continuous at  $x = c$  provided that  $f(c)$  exists,  $\lim f(x)$  exists, and  $\lim f(x) = f(c)$ .
- A function is continuous on an interval if the function is continuous at each point in the interval.
- Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous on all points in their domains.
- If the limit of a function exists at a discontinuity in its graph, then it is possible to remove the discontinuity by defining or redefining the value of the function at that point, so it equals the value of the limit of the function as  $x$  approaches that point.
- In order for a piecewise-defined function to be continuous at a boundary to the partition of its domain, the value of the expression defining the function on one side of the boundary must equal the value of the expression defining the other side of the boundary, as well as the value of the function at the boundary.
- The concept of a limit can be extended to include infinite limits.
- Asymptotic and unbounded behavior of functions can be described and explained using limits.
- The concept of a limit can be extended to include limits at infinity.
- Limits at infinity describe end behavior.
- Relative magnitudes of functions and their rates of change can be compared using limits.
- If  $f$  is a continuous function on the closed interval  $[a, b]$  and  $d$  is a number between  $f(a)$  and  $f(b)$ , then the Intermediate Value Theorem guarantees that there is at least one number  $c$  between  $a$  and  $b$ , such that  $f(c) = d$ .

## **Skills**

---

Students will be able to:

- Interpret the rate of change at an instant in terms of average rates of change over intervals containing that instant.
- Represent limits analytically using correct notation.
- Interpret limits expressed in analytic notation.
- Estimate limits of functions.
- Determine the limits of functions using limit theorems.
- Determine the limits of functions using equivalent expressions for the function or the squeeze theorem.
- Justify conclusions about continuity at a point using the definition.
- Determine intervals over which a function is continuous.
- Determine values of  $x$  or solve for parameters that make discontinuous functions continuous, if possible.
- Interpret the behavior of functions using limits involving infinity
- Explain the behavior of a function on an interval using the Intermediate Value Theorem.

## **Assessment and Resources**

---

### **School Formative Assessment Plan (Other Evidence)**

---

- Homework
- Quizzes
- Exit Tickets
- Reflections
- Performance Tasks

### **School Summative Assessment Plan**

---

- Unit Assessment

## **Primary Resources**

---

- Larson, Ron. Precalculus with Limits - A Graphing Approach. Houghton Mifflin Company, 2005

## **Supplementary Resources**

---

- IXL
- Kutasoftware
- Desmos
- Khan Academy
- PatrickJMP
- YouTube
- 

## **Technology Integration and Differentiated Instruction**

---

### **Technology Integration**

---

#### **Google Products**

- Google Classroom - Used for daily interactions with the students covering a vast majority of different educational resources (Daily Notes, Exit Tickets, Classroom Polls, Quick Checks, Additional Resources/ Support, Homework, etc.)
- GAFE (Google Apps For Education) - Using various programs connected with Google to collaborate within the district, co-teachers, grade level partner teacher, and with students to stay connected with the content that is covered within the topic. Used to collect data in real time and see results upon completion of the assignments to allow for 21st century learning.

#### **One to One Student's Laptop**

All students within the West Deptford School District are given a Chromebook, allowing for 21st century learning to occur within every lesson/topic.

#### **Graphing Calculators**

All students all required to utilize a graphing calculator, allowing for 21st century learning to occur within every lesson/topic.

## **Additional Support Videos**

Videos can be assigned from PatrickJMP, Kahn Academy, YouTube, etc. to support each of the lessons within this topic.

-

## **Differentiated Instruction**

---

### **Gifted Students (N.J.A.C.6A:8-3.1)**

- Students will complete Personal Progress Checks and will be assigned appropriate differentiated practice based on student performance.

### **English Language Learners (N.J.A.C.6A:15)**

- When discussing different ways of solving problems, focus on keywords and phrases. ELL Students might also benefit by using concrete objects to demonstrate different concepts.
- Create place cards or simple signs for students using the vocabulary words. They can use the signs as a reference throughout the lesson.
- Work with ELL Teacher to allow for all assignments to be completed with extra time.
- Pair ELL students with a student who is fluent in English.

### **At-Risk Students (N.J.A.C.6A:8-4.3c)**

- Students will complete Personal Progress Checks and will be assigned appropriate differentiated practice based on student performance.

### **Special Education Students (N.J.A.C.6A:8-3.1)**

- All other IEP modifications will be honored (ie. hard copies of notes, directions restated, etc.)
- Students will complete Personal Progress Checks and will be assigned appropriate differentiated practice based on student performance.

## **Interdisciplinary Connections**

---

**ELA** - Students will apply reasoning skills to justify statements. Students will justify statements through oral

and written communication.

**SCIENCE** - Students will analyze the distance, average velocity, and instantaneous velocity of objects.

**SOCIAL STUDIES** - Brief historical notes are found throughout both textbooks. They present stories of people and the research that they have done to advance the study of mathematics.

**WORLD LANGUAGES** -

**VISUAL/PERFORMING ARTS** -

**APPLIED TECHNOLOGY** -

**BUSINESS EDUCATION** -

**GLOBAL AWARENESS** -

## **Learning Plan / Pacing Guide**

---

### **HONORS TOPIC ONLY**

Week 1:

- Section 11.1 Limits of Functions Using Numerical and Graphical Techniques

Week 2:

- Section 11.2 Limits of Functions Using Properties of Limits
- Section 11.2 Review
- Section 11.2 Quiz

Week 3:

- Section 11.3 Continuity

Week 4:

- Section 11.5 Infinite Limits; Limits at Infinity; Asymptotes
- Section 11.5 Review
- Section 11.5 Quiz

Week 5:

- Unit Review
- Unit Test

